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# The Optimal Design of Social Security Benefits

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# **The Optimal Design of Social Security Benefits**

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## **Abstract**

The United States Social Security system is fairly unique in that it explicitly allows for a progressive formulation of retirement benefits by assigning a larger replacement rate to workers with small pre-retirement wages. In contrast, the public pension systems in other countries often replace a constant fraction of pre-retirement wages, although the length of the “averaging period” is typically shorter relative to the U.S. This paper examines the ex-ante optimal U.S. Social Security benefit structure using the model developed in Nishiyama and Smetters (2007). On one hand, progressivity in the benefit structure provides risk sharing against shocks that are difficult to insure privately. On the other hand, progressivity introduces various marginal tax rates that distort labor supply. Rather surprisingly, we find that the ex-ante best U.S. Social Security replacement rate structure is fairly “flat.” Intuitively, the relatively long averaging period used in the U.S. system formulation already provides some insurance against negative idiosyncratic shocks, but in a manner that is more efficient than explicit redistribution.

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## I. Introduction

This paper uses the model developed in Nishiyama and Smetters 2007 (herein NS2007) to derive the optimal structure of the U.S. Social Security system. Whereas NS2007 analyzed the efficiency of different, specific alternative designs of the U.S. Social Security system, the current paper attempts to derive the ex-ante best system design among a larger range of different potential Social Security formulations. This type of experiment naturally has many “degrees of freedom” that could potentially lead to a search over a massive parameter space. So we limit our experiments to (1) stationary fiscal policies; (2) pay-as-you-go financed systems and (3) a proportional payroll tax set equal to the current statutory value. These restrictions naturally allow us to focus on finding the ex-ante optimal steady-state function that translates previous earned wages into a Social Security benefit.

By “ex ante optimal,” we mean that we search for the best policy conditional on households not yet entering the workforce. Since these households have not yet realized their initial productive capacities, this perspective is sometimes called the “Rawlsian” experiment. In contrast, NS2007 analyzed Hick-sian efficiency gains or losses associated with different policies from an “interim” perspective where households have already realized their initial productivity types, which is highly correlated with the productivity earned throughout the rest of their lifecycle. The “ex ante” position, however, naturally places more emphasis on the importance of risk since even the initial household productivity type has not been materialized. So households in the ex-ante position should tend to care more about reducing that uncertainty relative to reducing the economic distortions associated policies that reduce that risk.

The current U.S. Social Security benefit formula is progressive in the sense that it gives a retiree who had relatively smaller average wages over his pre-retirement working years a larger replacement rate relative to a retiree who had a higher average wage rate. Very few other countries use a similar progressive formulation, although some countries with partially privatized systems provide a flat demogrant inside of the reduced public pension systems, which, in itself, is highly progressive. The U.S. Social Security system is also fairly unique in that it imputes a retiree’s pre-retirement average wage using a fairly long working history by selecting the worker’s “best 35” working years. In contrast, many other countries compute a worker’s average earnings across their wages earned just three to 10 years before retirement. To be sure, focusing on wages earned toward the end of the lifecycle tends to “bias upward” the worker’s computed average wage. But, it also places more risk on a worker’s uninsurable wages earned during that period; moreover, any *average* upward bias can be mostly undone by simply scaling down the replacement rate factor. In contrast, a longer average period itself reduces

some idiosyncratic risk but focuses more on riskiness of the earning process; it cannot be as easily reversed by scaling down the replacement rate factors.

But is all the progressivity in the U.S. Social Security system ex-ante optimal? On one hand, progressivity in the benefit structure provides risk sharing against shocks that are difficult to insure privately. On the other hand, progressivity introduces various marginal tax rates that distort labor supply.

Rather surprisingly, we find that the ex-ante best U.S. Social Security replacement rate structure is fairly “flat” – not very progressive – even though households face a considerable amount of uncertainty in the ex ante position. Intuitively, the relatively long averaging period used in the U.S. system formulation already provides sufficient insurance against negative idiosyncratic shocks but in a manner that creates lower labor supply distortions than a progressive benefit formula. Section II describes the model while Section III explains the calibration of the model. Section IV presents simulation results while Section V concludes.

## II. Model

Following NS2007, our model has three sectors: heterogeneous households with elastic labor supply; a competitive representative firm with constant-returns-to-scale production technology; and a government with a full commitment technology. We first describe the household sector before .

### II.A. The Household Sector

Households are heterogeneous with respect to the following variables: age  $i$ ; working ability  $e$  (measured by hourly wages); beginning-of-period wealth holdings  $a$ ; and, average historical earnings  $b$  that determine their Social Security benefits. Each year, a large number (normalized to unity) of new households of age 20 enter the economy. Population grows at a constant rate  $\nu$ . A household of age  $i$  observes an idiosyncratic working ability shock  $e$  at the beginning of each year and chooses its optimal consumption  $c$ , working hours  $h$ , and end-of-period wealth holding  $a'$ , taking as given the government’s policy schedule and future factor prices. At the end of each year, a fraction of households die according to standard mortality rates; no one lives beyond age 109. For simplicity, all households represent two-earner married couples of the same age.

Let  $\mathbf{s}$  denote the individual state of a household,

$$\mathbf{s} = (i, e, a, b),$$

where  $i \in I = \{20, \dots, 109\}$  is the household's age,  $e \in E = [e_{\min}, e_{\max}]$  is its age-dependent working ability (the hourly wage),  $a \in A = [a_{\min}, a_{\max}]$  is its beginning-of-period wealth, and  $b \in B = [b_{\min}, b_{\max}]$  is its average historical earnings for Social Security purposes.<sup>1</sup>

Let  $\mathbf{S}$  denote the state of the economy:

$$\mathbf{S} = (x(\mathbf{s}), W_G),$$

where  $x_t(\mathbf{s})$  is the joint distribution of households where  $\mathbf{s} \in I \times E \times A \times B$ .  $W_G$  is the net wealth of the rest of the government. Let  $\Psi$  denote the known government policy schedule:

$$\Psi = \{W_G, C_G, \tau_I(\cdot), \tau_P(\cdot), tr_{SS}(\mathbf{s}), tr_{LS}(\mathbf{s})\},$$

where  $C_{G,s}$  is government consumption,  $\tau_{I,s}(\cdot)$  is an income tax function,  $\tau_{P,s}(\cdot)$  is a payroll tax function for Social Security (OASDI) and  $tr_{SS,s}(\mathbf{s})$  is a Social Security benefit function. The specifications of these functions are described below.

The household's problem is described recursively as

$$(1) \quad v(\mathbf{s}, \mathbf{S}_t; \Psi_t) = \max_{c,h} u_i(c, h) + \beta(1 + \mu)^{\alpha(1-\gamma)} \phi_i E [v(\mathbf{s}', \mathbf{S}; \Psi) | e]$$

subject to

$$(2) \quad \begin{aligned} a' &= \frac{1}{1 + \mu} \{w \cdot eh + (1 + r)a - \tau_I(w \cdot eh, r \cdot a), tr_{SS}(\mathbf{s}) \\ &\quad - \tau_P(w \cdot eh) + tr_{SS}(\mathbf{s}) - c\} \geq a'_{\min}(\mathbf{s}), \\ a &= 0 \text{ if } i = 20, \quad a \geq 0 \text{ if } i \geq 65, \end{aligned}$$

where the utility function,  $u_i(\cdot)$ , takes the Cobb-Douglas form nested within a time-separable isoelastic specification,

$$(3) \quad u_i(c, h) = \frac{\{((1 + n_i/2)^{-\zeta} c)^\alpha (h_{\max} - h)^{1-\alpha}\}^{1-\gamma}}{1 - \gamma};$$

$\gamma$  is the coefficient of relative risk aversion;  $n_i$  is the number of dependent children at the parents' age  $i$ ;

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<sup>1</sup>The average historical earnings are used to calculate the Social Security benefits of each household. The variable  $b$  approximates the average indexed monthly earnings (AIME) multiplied by 12 as of age  $i$ .

$\zeta$  is the “adult equivalency scale” that converts the consumption by children into their adult equivalent amounts; and,  $h_{\max}$  is the maximum working hours. Wages are stochastic and follow a Markov process that is described in more detail below. In the “representative agent” economy setting described below, wages are assumed to be perfectly insurable; in the “heterogenous agent” case, wages are assumed to be uninsurable. We also consider an economy with and without the ability to annuitize household assets.

The constant  $\beta$  is the rate of time preference;  $\phi_i$  is the survival rate at the end of age  $i$ ;  $w$  is the wage rate per efficiency unit of labor (accordingly,  $w \cdot eh$  is total labor compensation at age  $i$ ); and  $r_t$  is the rate of return to capital. Individual variables of the model are normalized by the exogenous rate of labor augmenting technological change,  $\mu$ . Our choice for  $u_i(\cdot)$  is consistent with the conditions that are necessary for the existence of a long-run steady state in the presence of constant population growth. Hence,  $\mu$  is also equal to the per-capita growth rate of output and capital in steady state. The term  $\beta(1 + \mu)^{\alpha(1-\gamma)}$ , therefore, is the *growth-adjusted* rate of time preference.

$a'_{\min}(\mathbf{s})$  is the state-contingent minimum level of end-of-period wealth that is sustainable, that is, even if the household receives the worst possible shocks in future working abilities.<sup>2</sup> At the beginning of the next period, the state of this household when private annuity markets do not exist becomes

$$(4) \quad \mathbf{s}' = (i + 1, e', a' + q, b'),$$

where  $q_t$  denotes accidental bequests that a household receives at the end of the period.

The average historical earnings for this household,  $b$ , follows the following process,

$$(5) \quad b' = \begin{cases} 0 & \text{if } i \leq 24 \\ \frac{1}{i-24} \{ (i-25)b \frac{w_t}{w_{t-1}} + \min(w_t eh/2, weh_{\max,t}) \} & \text{if } 25 \leq i \leq 59, \\ b/(1 + \mu) & \text{if } i \geq 60 \end{cases},$$

where  $w \cdot eh_{\max}$  is the Old-Age, Survivors, and Disability Insurance (OASDI) tax cap, which is \$80,400 in 2001. U.S. Social Security benefits are computed on the basis of the highest 35 years of earnings. For simplicity, the model assumes that the highest 35 years of earnings correspond to ages between 25 and 59.

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<sup>2</sup>In particular,  $a'_{\min,t}(\mathbf{s})$  is allowed to be negative but cannot exceed in magnitude the present value of the worst possible future labor income stream at maximum working hours, sometimes called the “natural borrowing limit.” Although not shown explicitly in equation (2) in order to save on notation, any borrowing (i.e.,  $a' < 0$ ) by an agent age  $i$  at time  $t$  must be done at rate  $(1 + r_t)/\phi_i - 1$  in order to cover the chance that they will die before repaying their loan.

Let  $x(\mathbf{s})$  denote the population distribution of households, and let  $X(\mathbf{s})$  be the corresponding cumulative distribution. The distribution of households is adjusted by the steady-state population growth rate,  $\nu$ . The population of age 20 households is normalized to unity in the baseline economy along the balanced growth path, that is,

$$\int_E dX(20, e, 0, 0) = 1.$$

Let  $\mathbf{1}_{[a=y]}$  be an indicator function that returns 1 if  $a = y$  and 0 if  $a \neq y$ . Then, the law of motion of the measure of households is

$$x(\mathbf{s}') = \frac{\phi_i}{1 + \nu} \int_{E \times A \times B} \mathbf{1}_{[a' = a'(\mathbf{s}, \mathbf{S}; \boldsymbol{\Psi}) + q]} \mathbf{1}_{[b' = b'(w_t eh(\mathbf{s}, \mathbf{S}; \boldsymbol{\Psi}), b)]} \pi_{i,i+1}(e' | e) dX(\mathbf{s}),$$

where  $\pi_{i,i+1}$  denotes the transition probability of working ability from age  $i$  to age  $i + 1$ .

The *aggregate* value of accidental bequests each period is deterministic in our model because all risks are idiosyncratic and, therefore, uncorrelated across households. Accidental bequests could, therefore, be simply distributed equally and deterministically across all surviving households, as in previous work. That approach, however, suffers from two shortcomings. First, households would anticipate receiving a bequest with certainty, thereby artificially crowding out their pre-bequest savings. This savings reduction would be mitigated if bequests were random. Second, empirically, the inequality of bequests is important in generating a realistic measure of wealth inequality.

Our alternative strategy, therefore, distributes bequests randomly to surviving working-age households. Each household receives a bequest  $q$  with constant probability  $\eta$ :

$$q = \frac{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} a'(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t) dX(\mathbf{s})}{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} dX(\mathbf{s})},$$

$$\eta = \frac{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} dX(\mathbf{s})}{\sum_{i=20}^{64} \phi_i \int_{E \times A \times B} dX(\mathbf{s})}.$$

where  $q_t$  is the average wealth left by deceased households, and  $\eta$  is the ratio of deceased household to the surviving working-age households. In other words, a constant fraction  $\eta$  of households across all income groups will receive a bequest of size  $q_t$  while a constant fraction  $(1 - \eta)$  of households will not. But *ex ante*, each household only knows it will receive a bequest with probability  $\eta$ .<sup>3</sup>

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<sup>3</sup>Future work could go even a step further and allow for a correlation between a household's own income, age and the size of the bequest that they receive. At this point, however, we are not aware of any careful empirical work that would allow us to include this correlation into our model.



## II.B. Government

Government tax revenue consists of federal income tax  $T_I$ , and payroll tax for Social Security (OASDI)  $T_P$ . These revenues are

$$(6) \quad T_I = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_I(w \cdot eh(\mathbf{s}, \mathbf{S}; \Psi), r \cdot a, tr_{SS}(\mathbf{s})) dX(\mathbf{s}),$$

$$(7) \quad T_P = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_P(w \cdot eh(\mathbf{s}, \mathbf{S}; \Psi)) dX(\mathbf{s}).$$

Social Security (OASDI) benefit expenditure  $Tr_{SS}$  is

$$(8) \quad Tr_{SS} = \sum_{i=20}^{109} \int_{E \times A \times B} tr_{SS}(\mathbf{s}) dX(\mathbf{s}).$$

The law of motion of the government wealth (normalized by productivity growth and population growth) is

$$(9) \quad W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \{(1+r)W_{G,t} + T_I + T_P - Tr_{SS} - C_G\},$$

where  $C_G$  is government consumption and  $W_{G,t+1} = W_{G,t}$  in steady state.

## II.C. Measuring Ex-Ante Optimality

The ex-ante steady state welfare is given by

$$(10) \quad \Omega = \int_E v(20, e, 0, 0, \mathbf{S}; \Psi) dX(20, e, 0, 0)$$

Our experiments find the construction of Social Security benefits that maximizes the value of  $\Omega$ .

## II.D. Aggregation and Production

National wealth  $W_t$  is the sum of total private wealth, government net wealth  $W_G$ , and LSRA net

wealth  $W_{LS}$ ; and total labor supply  $L$  is measured in efficiency units:

$$(11) \quad W = \sum_{i=20}^{109} \int_{E \times A \times B} a \, dX(\mathbf{s}) + W_G,$$

$$(12) \quad L = \sum_{i=20}^{109} \int_{E \times A \times B} e \, h(\mathbf{s}, \mathbf{S}; \Psi) \, dX(\mathbf{s}).$$

In a closed economy, capital stock is equal to national wealth, that is,  $K = W$ , and gross national product  $Y$  is determined by a constant-returns-to-scale production function,

$$Y = F(K, L).$$

The profit-maximizing condition for this competitive firm is

$$(13) \quad F_K(K, L) = r + \delta,$$

$$(14) \quad F_L(K, L) = w,$$

where  $\delta$  is the depreciation rate of capital.

### *II.E. Recursive Competitive Equilibrium*

**Definition Recursive Competitive Equilibrium.** Let  $\mathbf{s} = (i, e, a, b)$  be the individual state of households, let  $\mathbf{S}_t = (x_t(\mathbf{s}), W_{LS,t}, W_{G,t})$  be the state of the economy, and let  $\Psi_t$  be the government policy schedule known at the beginning of year  $t$ ,

$$\Psi_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), tr_{SS,s}(\mathbf{s}), tr_{LS,s}(\mathbf{s})\}_{s=t}^{\infty}.$$

A series of factor prices  $\{r_s, w_s\}_{s=t}^{\infty}$ , accidental bequests  $\{q_s\}_{s=t}^{\infty}$ , the policy variables  $\{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, tr_{LS,s}(\mathbf{s})\}_{s=t}^{\infty}$ , the parameters of policy functions  $\{\varphi_s\}_{s=t}^{\infty}$ , the value function of households  $\{v(\mathbf{s}, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty}$ , the decision rule of households

$$\{d(\mathbf{s}, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty} = \{c(\mathbf{s}, \mathbf{S}_s; \Psi_s), h(\mathbf{s}, \mathbf{S}_s; \Psi_s), a'(\mathbf{s}, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty},$$

and the measure of households  $\{x_s(\mathbf{s})\}_{s=t}^{\infty}$ , are in a recursive competitive equilibrium if, in every period  $s = t, \dots, \infty$ , each household solves the utility maximization problem (1)–(5) taking  $\Psi_t$  as given; the

firm solves its profit maximization problem, the capital and labor market conditions (11)–(14) clear, and the government policy schedule satisfies (6)–(9). In steady-state,

$$\mathbf{S}_{t+1} = \mathbf{S}_t$$

for all  $t$  and  $\mathbf{s} \in I \times E \times A \times B$ .

### III. Calibration

#### III.A. Households

The coefficient of relative risk aversion,  $\gamma$ , is assumed to be 2.0. The number of dependent children at the parents' age  $i$ ,  $n_i$ , is calculated using the Panel Study of Income Dynamics (PSID) 2003 Family Data as shown in Table III. The "adult equivalency scale,"  $\zeta$ , is set at 0.6.<sup>4</sup> As discussed later,  $\beta$  is chosen to hit a target capital-output ratio that produces an interest rate of 6.25 percent in the initial steady state. The maximum working hours of husband and wife,  $h_{\max}$ , is set at 8,760, equal to 12 hours per day per person  $\times$  365 days  $\times$  two persons. A smaller value for  $h_{\max}$  would reduce the effective labor supply elasticity, and tend to reduce the gains from privatization. The parameter  $\alpha$  is chosen so that the average working hours of households between ages 20 and 64 equals 3,576 hours in the initial steady-state economy, the average number of hours supplied by married households in the 2003 PSID. Many of these parameters are summarized in Tables I and II. The parameters shown in Table I are the same for all of our privatization simulations. The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 2003 PSID, and is discussed in more detail in NS2007. Table IV shows the eight discrete levels of working abilities of five-year age cohorts. Using PSID, we estimate Markov transition matrixes that specify the probabilities that a household's wage will move from one level to a different level the next year. The population growth rate  $\nu$  is set to one percent per year while the survival rate  $\phi_i$  at the end of age  $i = \{20, \dots, 109\}$  are the weighted averages of the male and female survival rates, as calculated by the Social Security Administration [2001]. The survival rates at the end of age 109 are replaced by zero, thereby capping the maximum length of life.

#### III.B. Production

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<sup>4</sup>Hence, a married couple with two dependent children must consume about 52 percent (i.e.,  $2^{0.6} = 1.517$ ) more than a married couple with no children to attain the same level of utility, *ceteris paribus*.

Capital  $K$  is the sum of private fixed assets and government fixed assets. In 2000, private fixed assets were \$21,165 billion, government fixed assets were \$5,743 billion, and the public held about \$3,410 billion of government debt.<sup>5</sup> Government net wealth, therefore, is set equal to 9.5 percent of total private wealth in the initial steady-state economy. Moreover, the time preference parameter  $\beta$  is chosen in each variant of our model explored below so that the capital-GDP ratio in the initial steady state economy is 2.74, the empirical value in 2000.

Production takes the Cobb-Douglas form,

$$F(K, L) = AK^\theta L^{1-\theta}.$$

where, recall,  $L$  is the sum of working hours in efficiency units. The capital share of output is given by

$$\theta = 1 - \frac{\text{Compensation of Employees} + (1 - \theta) \times \text{Proprietors' Income}}{\text{National Income} + \text{Consumption of Fixed Capital}}.$$

The value of  $\theta$  in 2000 was 0.30.<sup>6</sup> The annual per-capita growth rate  $\mu$  is assumed to be 1.8 percent, the average rate between 1869 to 1996 (Barro, 1997). Total factor productivity  $A$  is set at 0.949, which normalizes the wage (per efficient labor unit) to unity.

The depreciation rate of fixed capital  $\delta$  is chosen by the following steady-state condition,

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \mu - \nu.$$

In 2000, private gross fixed investment accounted for 17.2 percent of GDP, and government (federal and state) gross investment accounted for 3.3 percent of GDP.<sup>7</sup> With a capital-output ratio of 2.74, the ratio of gross investment to fixed capital is 7.5 percent. Subtracting productivity and population growth rates, the annual depreciation rate is 4.7 percent.

### *III.C. The Government*

Federal income tax and state and local taxes are assumed to be at the level in year 2001 before the passage of the “Economic Growth and Tax Relief Reconciliation Act of 2001” (EGTRRA). Since households in our model are assumed to be married, we use a standard deduction of \$7,600. However,

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<sup>5</sup>Source: Department of Commerce, Bureau of Economic Analysis.

<sup>6</sup>*Ibid.*

<sup>7</sup>*Ibid.*

following Altig *et al.* [2001], we allow higher income households to itemize deductions when it is more valuable to do so, and we assume that the value of the itemized deduction increases linearly in the Adjusted Gross Income.<sup>8</sup> The additional exemption per dependent person is \$2,900 where the number of dependent children is consistent with Table III. Table V shows the statutory marginal tax rates before EGTRRA.<sup>9</sup> As noted earlier, a household's labor income in this calibration includes the imputed payroll tax paid by its employer. Thus, taxable income is obtained by subtracting the employer portion of payroll tax from labor income.

The standard deduction, the personal exemption, and all tax brackets grow with productivity over time so that there is no real bracket creep; this indexing is also needed for the initial economy to be in steady state. Since the effective tax rate on capital income is reduced by investment tax incentives, accelerated depreciation and other factors [Auerbach 1996], the tax function is further adjusted so that the cross-sectional average tax rate on capital income is about 25 percent lower than the average tax on labor income.<sup>10</sup> In 2000, the ratio of total individual federal income tax revenue (not including Social Security and Medicare taxes) to GDP was 0.102 and the ratio of corporate income tax to GDP was 0.021. Each statutory federal income tax rate shown in Table V, therefore, is multiplied by  $\varphi_I$  so that tax revenue (including corporate income tax) totals 12.3 percent of GDP in the initial steady state. The adjustment factor is 0.815. State and local income taxes are modeled parsimoniously with a 4.0 percent flat tax on income above the deduction and exemption levels used at the federal level.

The tax rate levied on employees for Old-Age, Survivors, and Disability Insurance (OASDI) is 12.4 percent, and the tax rate for Medicare (HI) is 2.9 percent. In 2001, employee compensation above \$80,400 was not taxable for OASDI. (See Table VI.) Workers with wages above \$80,400, therefore, don't face a marginal tax or distortions from the Social Security system.

Social Security benefits are based on each worker's Average Indexed Monthly Earnings (AIME),  $b/12$ , and the replacement rate schedule shown in Table VII. The replacement rates are 90 percent for the first \$561, 32 percent for amounts between \$561 and \$3,381, and 15 percent for amounts above \$3,381. Social Security, therefore, is progressive in that a worker's benefit relative to AIME (the "replacement rate") is decreasing in the AIME.

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<sup>8</sup>In particular, the deduction taken by a household is the greater of the standard deduction and  $0.0755 \times \text{AGI}$ , or  $\max\{\$7600, 0.0755 \times \text{AGI}\}$ .

<sup>9</sup>The key qualitative results reported herein are unaffected if the tax function were instead modeled as net taxes, that is, after subtracting transfers indicated in the Statistics of Income.

<sup>10</sup>This relative reduction to the tax rate on capital is commonly used by the Congressional Budget Office, and it balances the legal tax preferences given to capital versus the legal tax benefits given to labor, including tax-preferred fringe benefits.

The U.S. OASDI also pays spousal, survivors' and disability benefits in addition to the standard retirement benefit described above. Indeed, retiree benefits accounted for only 69.1 percent of total OASDI benefits in December 2000.<sup>11</sup> OASDI benefits, therefore, are adjusted upward by the proportional adjustment factor  $\varphi_{SS}$  so that total benefit payments equal total payroll tax revenue. The adjustment factor  $\varphi_{SS}$  equals about 1.46 in our model. This adjustment proportionally distributes non-retiree OASDI payments across retirees.

#### IV. Policy Experiments

We now examine the impact of changing the progressivity of the Social Security benefit on various steady-state macro-economic variables as well as ex-ante efficiency. We assume that private annuity markets do not exist, and so Social Security provides risk sharing over both uninsurable idiosyncratic wages earned over the household's lifetime as well as annuity protection. The change in welfare that we report is calculated by reporting the value of  $\left[\frac{\Omega^{NEW}}{\Omega^0}\right]^{1/(1-\gamma)}$ , where  $\Omega^0$  represents the initial steady-state ex-ante welfare and  $\Omega^{NEW}$  is the welfare after the change in the social security benefit function. Scaling by  $1/(1-\gamma)$  allows us to interpret the induced welfare changes as the percentage change in full (potential) lifetime income for the agent in the ex-ante (pre-working) position.

The results are reported in Table VIII. Run 1 corresponds to the current baseline, and so there are no change in any variable, by definition. The remaining runs then alter the Social Security replacement rate for the first \$561 from 90% to different values: 45% (Run 2), 135% (Run 3) and 180% (Run 4). We pay special attention to this first "bendpoint" in the replacement rate function because it effectively controls most of Social Security's progressivity under the balanced-budget constraint that keeps the payroll tax rate constant. In particular, the larger the value of the first bendpoint, the smaller the other bendpoints must become in order to achieve pay-as-you-go balance with the current payroll tax rate. So, in effect, the first bendpoint alone largely determines the progressivity of the entire benefit formula.

However, any policy change also produces additional general-equilibrium effects through factor prices, including interest rates and wages. To maintain a given level of spending in the federal system outside of Social Security, which is not a consumption good in our model, the federal income tax rate is change proportionally, as shown in Table VIII. To maintain pay-as-you-go balance inside of Social Security with a constant tax rate, however, we adjust Social Security benefits upward or downward in a proportional manner across all households. As a result, some of the direct change in the first bendpoint

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<sup>11</sup>See Table 5.A1 in Social Security Administration [2001].

is slightly offset; for example, for Run 2, the first bendpoint adjusts from 45% to 56.4% in light of these general-equilibrium offsets. But this approach – rather than changing payroll tax rates – allows for the cleanest interpretation of the results since these bendpoint parameters still fundamentally alter the progressivity of the system without altering the statutory rate.

Run 2 shows the effects of a reduction in progressivity relative to the current Social Security system. National wealth and labor supply both increase, as does ex-ante welfare. In contrast, Runs 3 and 4 consider more progressive benefit formulas relative to the current system. In Run 3, which increases progressivity just a modest amount, national wealth and labor supply both decrease, as does ex ante welfare. In Run 4, where progressivity is increased even more, the losses become even larger in magnitude. Taken together, these results suggest a fairly monotonic relationship between ex-ante welfare and progressivity: a less progressive benefit structure leads to higher ex-ante welfare. These results are robust to using a more “fine grid” over the bendpoint parameters as well as somewhat different methods of adjusting the second and third bendpoints.

The ex-ante superiority of the relatively flatter benefit schedule is surprising at first glance, especially when welfare is measured from the ex-ante position. There are two competition effects taking place. On one hand, greater amount of progressivity in the benefit formula pools more of the uncertainty that households face, including their initial productivity allocation that is highly correlated with their subsequent wages. Since these risks are otherwise uninsurable in the private market, a flatter benefit formula reduces some of this valuable risk sharing. On the other hand, progressivity produces complicated distortions to labor supply by effectively subsidizing labor supply at lower wages while increasing effective marginal tax rates at higher wages. The effects of these distortions appear to outweigh the benefits of pooling wage uncertainty.

Intuitively, the Social Security benefit calculation already provides some limited risk sharing even without progressive benefits, simply due to the long computation window that is used to calculate a person’s pre-retirement wage; low wage realizations can be balanced against higher wage realizations. While this averaging does little to insure against the uncertainty in a household’s initial productivity allocation, it still provides some insurance value over the lifecycle without producing the same effective tax rates as explicit progressivity, much like how precautionary saving works. Adding additional progressivity with distortions does not seem to not create enough value despite the additional risk sharing it provides.

## VII. Concluding Remarks

While partial privatization of the U.S. Social Security system is likely “off the table” for the foreseeable future, significant projected shortfalls still remain in the Social Security program. Tough choices, therefore, will have to be made regarding increasing revenue and/or decreasing benefits. We have not explicitly modeled the expected shortfalls in this paper; indeed, such imbalances are generally not consistent with standard rational expectations models where budget constraints are assumed to hold. But, our analysis does suggest that some caution is in order for reforms that focus heavily on making the benefit structure even more progressive, such as “progressive price indexing,” which has been a quite popular option debated during the past couple years. Other features of the U.S. – namely, the large computation period – naturally provide risk sharing but in a less distorting manner.



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### References

Altig, David, Alan Auerbach, Laurence Kotlikoff, Kent Smetters, and Jan Walliser, "Simulating Fundamental Tax Reform in the United States," *American Economic Review*, XC1 (2001), 574-595.

Auerbach, Alan, "Tax Reform, Capital Allocation, Efficiency and Growth," in H. Aaron and W. Gale, eds, *Economic Effects of Fundamental Tax Reform* (Washington, D.C.: Brookings Institution Press, 1996).

Barro, Robert J, *Macroeconomics (5th edition)* (Cambridge, MA and London England: The MIT Press, 1997).

Liebman, Jeffrey B., "Redistributional in the Current U.S. Social Security System," in Martin Feldstein and Jeffrey B. Liebman, eds, *The distributional aspects of Social Security and Social Security reform* (Chicago and London: University of Chicago Press, 2002), 11-41.

Nishiyama, Shinichi, "Analyzing an Aging Population—A Dynamic General Equilibrium Approach," Congressional Budget Office, Working Paper 2004-3 (2004).

Nishiyama, Shinichi and Kent Smetters, "Does Social Security Privatization Produce Efficiency Gains," *Quarterly Journal of Economics*, November, 2007: 1677-1719.

Social Security Administration, *Social Security Bulletin: Annual Statistical Supplement*, available at <http://www.ssa.gov/statistics/supplement/2001/index.html> (2001).

TABLE I  
PARAMETERS INDEPENDENT OF MODEL ASSUMPTIONS

Coefficient of relative risk aversion	$\gamma$	2.0
Capital share of output	$\theta$	0.30
Depreciation rate of capital stock	$\delta$	0.047
Long-term real growth rate	$\mu$	0.018
Population growth rate	$\nu$	0.010
Probability of receiving bequests	$\eta$	0.0161
Total factor productivity <sup>a</sup>	$A$	0.949

*a.* Total factor productivity is chosen so that  $w$  equals 1.0.

TABLE II  
PARAMETERS THAT VARY BY MODEL ASSUMPTIONS

Time preference <sup>a</sup>	$\beta$	0.981
Consumption share <sup>b</sup>	$\alpha$	0.501
Income tax adjustment <sup>c</sup>	$\varphi_I$	0.815
OASDI benefit adjustment <sup>d</sup>	$\varphi_{SS}$	1.456

*a.* The capital-GDP ratio is targeted to be 2.74 ( $r = 6.25$  percent) without annuity markets.

*b.* The average annual working hours are 3,576 per married couple when  $h_{\max} = 8,760$ .

*c.* In a heterogeneous-agent economy, the ratio of income tax revenue to GDP is 0.123.

*d.* OASDI benefits are pay-as-you-go.

TABLE III  
NUMBER OF PEOPLE UNDER AGE 18 LIVING IN A MARRIED HOUSEHOLD

Age cohorts	Number of children	Age cohorts	Number of children
20-24	0.824	50-54	0.576
25-29	0.957	55-59	0.196
30-34	1.512	60-64	0.109
35-39	1.759	65-69	0.084
40-44	1.700	70-74	0.025
45-49	1.152	75-79	0.028

Source: Authors' calculations from the 2003 Panel Study of Income Dynamics (PSID).

TABLE IV  
WORKING ABILITIES OF A HOUSEHOLD (IN U.S. DOLLARS PER HOUR)

Percentile		Age cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
$e^1$	0-20th	6.59	6.79	7.23	7.58	6.63	7.06
$e^2$	20-40th	9.13	11.90	12.99	14.13	13.65	14.01
$e^3$	40-60th	11.13	15.15	17.63	19.43	18.76	19.84
$e^4$	60-80th	13.89	18.79	23.72	25.98	26.56	26.51
$e^5$	80-90th	17.89	23.07	31.94	36.66	37.30	34.38
$e^6$	90-95th	22.17	28.75	44.87	50.36	51.30	43.69
$e^7$	95-99th	28.92	37.02	70.45	90.33	74.86	76.14
$e^8$	99-100th	50.99	68.56	111.40	180.53	211.09	239.59
Percentile		Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
$e^1$	0-20th	6.45	2.76	0.02	0.00	0.00	0.00
$e^2$	20-40th	14.02	11.90	4.54	0.01	0.00	0.00
$e^3$	40-60th	20.46	17.75	12.55	3.56	0.00	0.00
$e^4$	60-80th	27.89	25.24	20.40	12.35	1.64	0.35
$e^5$	80-90th	37.71	32.90	32.30	22.41	7.45	10.15
$e^6$	90-95th	47.60	43.79	42.47	34.78	12.52	20.57
$e^7$	95-99th	81.61	68.69	57.48	47.01	19.22	36.73
$e^8$	99-100th	247.47	443.14	89.02	101.28	100.08	51.30

Source: Authors' calculations from the 2003 PSID.

TABLE V  
MARGINAL INDIVIDUAL INCOME TAX RATES IN 2001 (MARRIED HOUSEHOLD, FILED JOINTLY)

Taxable income			Marginal income tax rate (%)
\$0	–	\$45,200	$15.0 \times \varphi_I$
\$45,200	–	\$109,250	$28.0 \times \varphi_I$
\$109,250	–	\$166,500	$31.0 \times \varphi_I$
\$166,500	–	\$297,350	$36.0 \times \varphi_I$
\$297,350	–		$39.6 \times \varphi_I$

TABLE VI  
MARGINAL PAYROLL TAX RATES IN 2001

Taxable labor income per worker			Marginal tax rate (%)	
			OASDI	HI
\$0	–	\$80,400	$12.4 \times \varphi_P$	2.9
\$80,400	–		$0.0 \times \varphi_P$	2.9

Note: The payroll tax adjustment factor  $\varphi_P$  equals 1.0 in the baseline economy.

TABLE VII  
OASDI REPLACEMENT RATES IN 2001

AIME (b/12)			Marginal replacement rate (%)
\$0	–	\$561	$90.0 \times \varphi_{SS}$
\$561	–	\$3,381	$32.0 \times \varphi_{SS}$
\$3,381	–		$15.0 \times \varphi_{SS}$

Note: The OASDI benefit adjustment factor  $\varphi_{SS}$  is set so that the OASDI is pay-as-you-go in the baseline economies.

TABLE VIII  
PERCENT CHANGE IN SELECTED STEADY-STATE MACRO VARIABLES AND WELFARE RELATIVE  
TO BASELINE

Run # <sup>a</sup>	GNP	National wealth	Labor supply	Interest rate	Wage rate	Income tax rate <sup>b</sup>	Ex-Ante Welfare
1 Bendpoints 90 / 32 / 15	0	0	0	0	0	0	0
2 Bendpoints 56.4 / 40.1 / 18.8	0.55	0.45	0.60	0.18	-0.04	-0.95	0.04
3 Bendpoints 112.2 / 26.6 / 12.5	-0.36	-0.26	-0.41	-0.18	0.04	0.65	-0.02
4 Bendpoints 127.8 / 22.7 / 10.7	-0.63	-0.45	-0.70	-0.30	0.07	1.11	-0.04

*a.* Closed economy, no private annuity markets.

*b.* The proportional change in marginal tax rates across all households.